

TMS 2.47 CONSIDER A PARTICLE IN  $x > 0$  UNDER

$$U(x) = U_0 \left( \frac{\alpha}{x} + \frac{x}{\alpha} \right)$$

WHERE  $U_0 = 1\text{J}$  AND  $\alpha = 2\text{m}$ . PLOT THE POTENTIAL, FIND THE EQUILIBRIUM POINTS & DETERMINE THEIR STABILITY.

FIND EQUILIBRIUM

$$\left. \frac{dU}{dx} \right|_{x_0} = U_0 \left( -\frac{\alpha}{x^2} + \frac{1}{\alpha} \right) \Big|_{x_0} = 0$$

$$\frac{\alpha}{x_0^2} = \frac{1}{\alpha}$$

$$x_0^2 = \alpha^2 \Rightarrow \boxed{x_0 = \alpha} \Rightarrow x_0 = 2$$

FIND STABILITY

$$\left. \frac{d^2U}{dx^2} \right|_{x=x_0} = \frac{d}{dx} \left( \frac{1}{\alpha} - \frac{\alpha}{x^2} \right) \Big|_{x=x_0}$$

$$= 0 + \frac{2\alpha}{x^3} \Big|_{x=x_0}$$

$$= \frac{2\alpha}{\alpha^3} = \frac{2}{\alpha^2} > 0 \Rightarrow \text{STABLE EQUILIBRIUM}$$